Differentiation

in terms of x

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Differentiation in terms of x

Instructions

1. Rearrange the equation, use the Laws of Indices to do all expansions and factorising to obtain the equation of the form $y = ax^n + \cdots$
2. Use the formula $\frac{dy}{dx} = anx^{n-1}$ for each term to determine their derivative.
3. Check to see if the terms can be simplified.

Example What is the gradient of $y = 6x^3\left(\frac{5}{x} + \sqrt{4x} - (3x^3)^4\right)$ at x=3?

$$y = 6x^3\left(\frac{5}{x} + \sqrt{4x} - (3x^3)^4\right)$$

$$= 6x^3\left(5x^{-1} + 2x^{\frac{1}{2}} - 81x^{12}\right)$$

$$= 30x^2 + 12x^{\frac{7}{2}} - 486x^{15}$$

$$\therefore \frac{dy}{dx} = 60x + 42x^{\frac{5}{2}} - 7290x^{14}$$

$$= 60(3) + 42(3)^{\frac{5}{2}} - 7290(3)^{14}$$

$$= 180 + 42(9\sqrt{3}) - 7290(478969)$$

$$= 180 + 378\sqrt{3} - 3{,}491{,}684{,}010$$

$$= 378\sqrt{3} - 3{,}491{,}683{,}830$$

Questions

For each of these questions, find the value of the gradient at x=2

1 $y = 3x^2$

2 $3y = 9x^4$

3 $5x^4 = 24 - y$

4 $y = \sqrt{x}$

5 $y = x\sqrt{x}$

6 $2y = x\sqrt{x}$

7 $y = 4x^3 + (2x^2)^4$

8 $y = 5$

9 $3y = 7x^3 + 12$

10 $y = 5x^2\left(\frac{x}{15} + 6x\right)$

11 $x^2 + y = 35$

12 $y = \frac{5}{x^3} - \frac{2}{x^2}$

13 $y = \frac{6}{x^5} + \frac{2}{x^3}$

14 $6x = 3y - x^2$

15 $y = \frac{2}{3}x^2\left(x^3 - (3x^2)^4 + \sqrt{x}\right)^3$

16 A cuboid has edges $5x, 6x\ and\ 3x$.

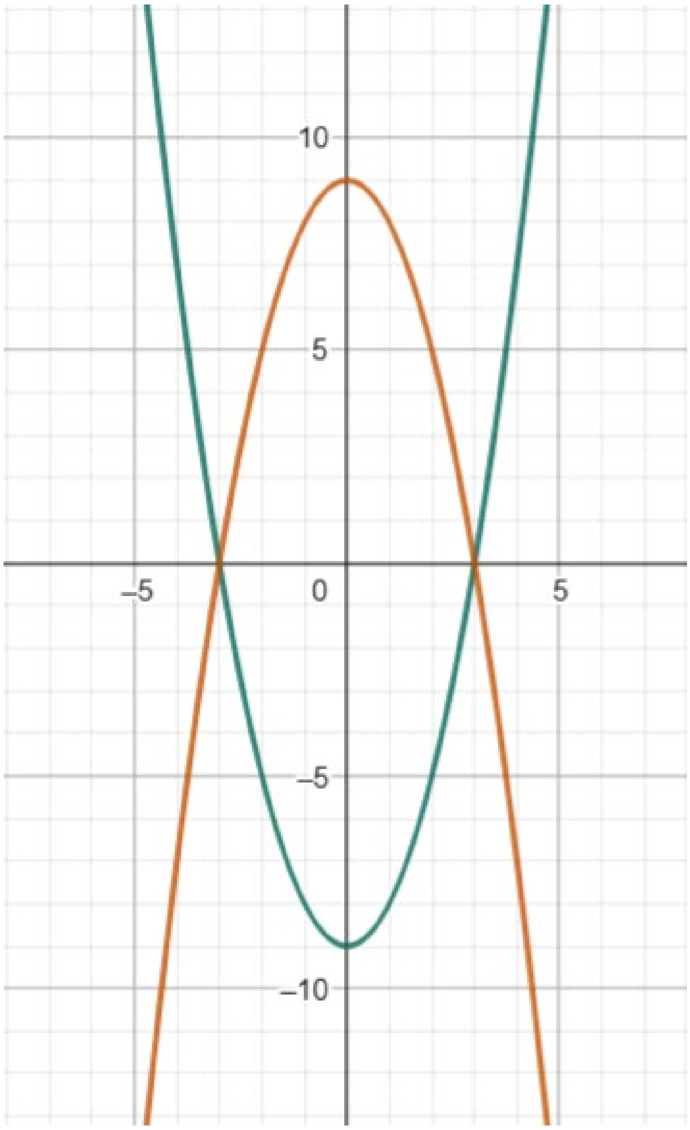
a Find the surface area A and the volume V of the cuboid in terms of x.

b Calculate $\frac{dA}{dx}$ and $\frac{dV}{dx}$.

17 Calculate the co-ordinates on the curve $y = 3x^3 - 2x^2 + 8x - 9$ where the gradient is equal to zero.

18 Find the gradient of the curve $y = 2x\sqrt{x} + \frac{2}{\sqrt{x}}$ at the point (1,4)

19 The graphs below show $y = 9 - x^2$ and $y = x^2 - 9$



a For each graph, work out the gradient of the curve at points $x = -2$ and $x = 2$.

b Write the equations of the tangents to the curves at those four points.

c Add the tangents to the diagram above and state the points of intersection of the tangents with the x and y axes.

d What type of quadrilateral is formed?

20 For the function $y = x^2 - x - 2$:

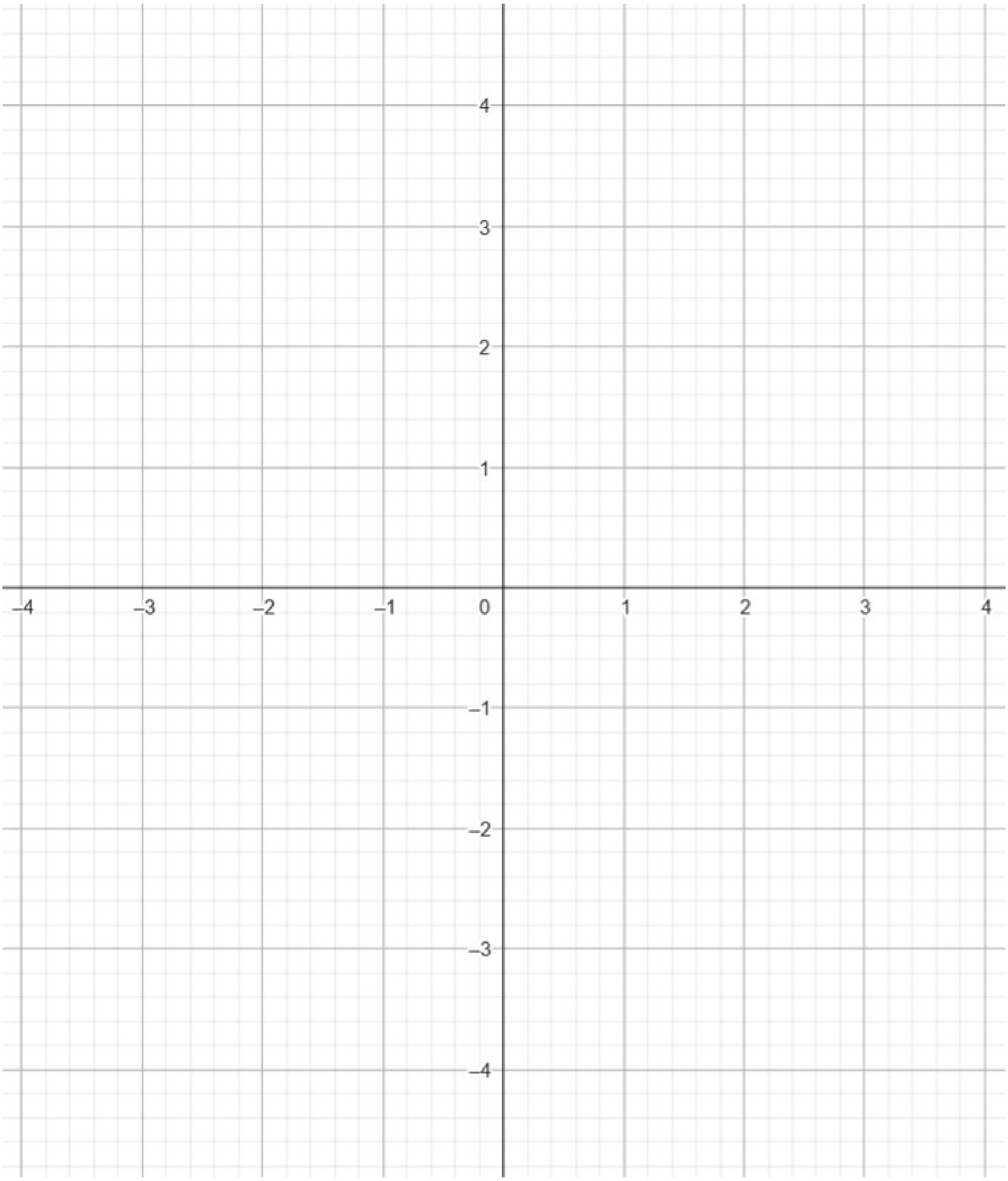
a Calculate $\frac{dy}{dx}$ and the values of x for which $\frac{dy}{dx} = 0$.

b Find the value of $\frac{d^2y}{dx^2}$ at those points.

c State whether these points are maxima or minima.

d Work out the co-ordinates of these points.

e Sketch the curve.



21 Write down the formula for finding the volume of a sphere.

Find the term for $\frac{dV}{dr}$.

What have you found?

Motion with variable acceleration

When you have constant acceleration, you can use Newton's SUVAT equations to calculate the velocity, displacement, time etc. However, that does not work if you have variable acceleration. To solve these, you must use Calculus.

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

a=acceleration, v=velocity, s=displacement, t=time

22 For the first five seconds, the displacement of a sports car from its original position is given by the function

$$s = 8t^2 - t^3$$

a Find an expression for the velocity in terms of t.

b Find the initial velocity.

c Find the velocity after 5 seconds.

d Find an expression for the acceleration in terms of t.

e Find the acceleration after 6 seconds.

f On a motorway, the speed limit is 70mph. After 5 seconds, is the driver breaking the law? $1\ km \approx \frac{5}{8}\ mile$

23 Will is a fighter pilot. He flies an 11,000kg fighter jet which has a take-off speed of 185 mph and needs to reach this within 8 seconds of starting to move. In this time, the displacement can be modelled by the function

$$s = 12t^3 - t^4$$

a Is the aircraft going fast enough to take off after 8 seconds?

b Bearing in mind, $F = ma$, what is the thrust of the fighter jet in the first 8 seconds?

24 A ball is launched vertically and its height, h metres, above the ground after t seconds is given by the function

$$h = 2 + 7t - t^2$$

a From what height was the ball thrown?

b Find an expression for the velocity of the ball at time t.

c When is the ball instantaneously at rest?

d What is the maximum height that the ball reaches?

e Bill measures the time from the ball's launch to when it hits the ground. What is the final reading on his stopwatch?

f At what speed is the ball travelling as it hits the ground?